# SYNCHRONIZATION IN ENSEMBLES OF PHASE-LOCKING SYSTEMS WITH UNIDIRECTIONAL COUPLING

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Phase-locking systems (PLS) are very widespread all over the branches of industry and fields of science. Radiolocation, energy industry, atomic clock production are hardly imaginable without PLS usage. Abstractedness of the theoretical model of PLS is essential for the description of various biological, mechanical and other oscillatory systems.

Let us consider the base model of PLS – the ensemble of phase-locking loops without frequency filter, coupled unidirectionally in the control circuit. The equations that describe this system are ([1]):

$$\left| \frac{d\varphi_1}{d\tau} = \gamma - \sin(\varphi_1) \right|,$$

$$\left| \frac{d\varphi_n}{d\tau} = \gamma - \sin(\varphi_n) - \delta \sin(\varphi_{n-1}) \right|,$$
(1)

where  $\gamma$  is a frequency detuning,  $\delta$  – coupling parameter,  $\varphi_i$  – current phase of its PLL, n – number of PLL, and n takes integer values from 2 to N – total number of elements in ensemble.

There is the symmetry of the ODE system above – system (1) transforms to itself with the following transform: all the coordinates and frequency detuning are multiplied by -1. This means that to study system dynamics is enough only for positive values of  $\gamma$ . The fact, that all coordinates are presented in the equations with the  $2\pi$ -periodic functions means that the phase space of system (1) has *n*-dimensional torus topology.

Dynamics of PLS without frequency filter for positive values of  $\delta$  has been studied [2], therefore accurate analysis of synchronous regimes of PLS containing many PLL's is of great interest for all real values of  $\delta$ . Existence conditions for the equilibrium states of system (1) could be found by means of standard methods of theory of oscillations that lead to a system of inequalities that describes the boundaries of synchronization areas. The intersection of these areas is the area of global synchronization of initial interest. Hence, for each element one can obtain the following progression of inequalities:

$$1 - \frac{1}{\gamma} < \sum_{n=1}^{N-1} (-\delta)^n < 1 + \frac{1}{\gamma}$$
(2)

On the Fig. synchronization areas  $C_S^N$  of the first eight elements are presented. Boundaries in the positive part of axis  $\delta$  have some fractures in the point "a" lying on the line  $\gamma = \delta - 1$ . In negative part of axis  $\delta$  the boundaries of areas  $C_S^N$  are smooth on the interval  $0 < \gamma < 1$ . When  $\gamma$  is greater than 1 the first element has nonzero phase velocity in every point of phase space, so there is no global synchronization regimes with such values of  $\gamma$ .



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# DYNAMICS OF ELECTRONIC NEURONS COUPLED VIA OPTICAL FIBER CHANNEL

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A design of electronic neuron networks capable to reproduce brain functions in silico is one of the most intriguing challenges in modern science and engineering. Such systems give an opportunity to develop new generation of information processing technologies based on brain computation principles. Another interesting application is to create an interface between electronic circuits and living neurons for biomedical applications.

In this work we propose model of interaction between synaptically coupled electronic neurons, where the optical fiber imitates a neuron axon. Each neuron is implemented as a pulse signal generator based on FitzHugh-Nagumo system. After testing and tuning the presynaptic and postsynaptic neuron oscillators, we connect them by the optical fiber. Electronic circuit of optically coupled electronic neurons illustrated in this slide. The output signal from the postsynaptic neuron is conveyed to the light emitted by a diode and sent into the fiber. At the end of the fiber the signal detected by a photodiode enters to the postsynaptic neuron. Such unidirectional signal transmission simulates the functionality of an excitatory synaptic coupling.

After testing and tuning the presynaptic and postsynaptic neuron oscillators, we connect them by the optical fiber. The output signal from the postsynaptic neuron is conveyed to the light emitted by a diode and sent into the fiber. At the end of the fiber the signal detected by a photodiode enters to the postsynaptic neuron. Such unidirectional signal transmission simulates the functionality of an excitatory synaptic coupling.

It is given that the energy loss in the optical fiber is negligible, and then the strength of the synaptic coupling is only determined by the photodetector sensitivity at the end of the optical fiber. An increase of this sensitivity provides forced synchronization of the postsynaptic neuron via the optoelectronic coupling. By varying the sensitivity of the photodiode for load resistance of 500 kOm to 1000 kOm the synchronization regimes with 1:1 and 3:1 frequency-locking ratios are observed. We also find that low sensitive photodiodes with load resistance of 1 kOm to 500 kOm are not capable to provide synchronization.

So we demonstrated experimentally frequency-locking regimes of the postsynaptic neuron forced by the presynaptic neuron to different 1:N ratios (where N – is the number of spikes of the postsynaptic neuron to every spike of presynaptic neuron).

# HIGH PERFORMANCE COMPUTING FOR MODELING DYNAMICS OF BIOLOGICALLY REALISTIC NEURAL NETWORKS

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One of the very interesting issues in modern neuroscience is studying signaling processes of cultured neural networks cultured on Multi-Electrode Arrays (MEA). Despite their apparent simplicity, cultured neural networks show intrinsic types of population activity typical for the intact brain, e.g. bursting and epileptiform activity [1, 2].

It is well known, however, that making experiments on any living tissue is associated with many limitations: they are extremely expensive and difficult to handle and maintain. Studying neural networks *in vitro* is not an exception. Computational modeling is a widely used approach aimed to overcome these limitations. In our work we presented a computer program implementing high performance calculations for modeling activity of biologically realistic neuronal network with fine-tuned parameters for reproducing activity of experimentally observed living neuronal network *in vitro*. In our model the dynamics of a single neuron was described by Izhikevich model (1) [3].

$$\begin{cases} C_m \overset{\Box}{V} = k(V - V_r)(V - V_l) - U + I + I_{syn} \\ \overset{\Box}{U} = a(b(V - V_r) - U) \\ if \quad V \ge V_{thr}, \quad then \\ \begin{cases} V = c \\ U = U + d \end{cases} \end{cases}$$
(1)

The single neuron model is characterized by its both, mathematical simplicity and biological plausibility.

Modeling networks composed of interconnected neurons also requires using a simple and effective model of a synapse. For this purpose we used Tsodyks-Uziel-Markram model (TUM model) (2) allowing to mimic short term synaptic plasticity phenomenon [4].

$$\begin{cases} \frac{dx}{dt} = \frac{z}{\tau_{rec}} - ux\delta(t - (t_{sp} + d)) \\ \frac{dy}{dt} = -\frac{y}{\tau_{psc}} + ux\delta(t - (t_{sp} + d)) \\ \frac{dz}{dt} = \frac{y}{\tau_{psc}} - \frac{z}{\tau_{rec}} \\ \frac{du}{dt} = -\frac{u}{\tau_{fac}} + V(1 - u)\delta(t - (t_{sp} + d)) \\ I_{syn}(t) = \sum_{i} W_{i} * y_{i}(t) * (V - V_{rev}) \end{cases}$$

$$(2)$$

Being a high dimensional nonlinear dynamical system our model has to be implemented using high performance computing approach. For accelerating computations the CUDA technology was used. This technology allows to perform calculations using graphical processing units (GPU). Each GPU consists of many calculating cores capable to execute many parallel threads simultaneously.

For communicating between the neurons the only times of the spikes emitted by each neuron was used. Thus when there aren't any input spikes on neuron we can run calculating it's dynamics independently from others. CUDA is known to run faster when calculations performed by every thread are independent, because there are no costs for data transmission and synchronization between the threads. Therefore using CUDA we can significantly increase the computing performance. Our simulated model consisted of 500 neurons and 2500 synapses with CUDA was from 4 to 8 times faster, compared to usual CPU calculations. The results of the simulations are shown on figure. Here the activity of the whole network is depicted in a form of a raster plot. On the *x* axis there is the time. On the *y* axis of top plot, there is the index of a neuron, and the spike timings are depicted by dots. On the *y* axis of the bottom plot, there is the total network activity (number of spikes in 1 ms).



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## CHERENKOV-TYPE TERAHERTZ EMISSION SPECTROSCOPY FOR EXPLORING ULTRAFAST INVERSE FARADAY EFFECT

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The interaction of subpicosecond laser pulses with magnetic materials is of great interest both for fundamental researches and applied technologies [1]. This interest was triggered by the first demonstration of the ultrafast light control of magnetization in 1996 [2]. It is generally accepted to divide optomagnetic phenomena into thermal and nonthermal. Non-thermal effects that do not require the absorption of photons are attributed to the inverse Faraday effect (IFE). This effect consists in generation of magnetization in magnetic media by circularly polarized light. The IFE was theoretically predicted [3] and experimentally observed for 30 ns laser pulses [4] more than 50 years ago. In recent paper [5] the IFE was demonstrated on the subpicosecond time scale, but the mechanism of the effect for the ultrashort pump pulses is still under intense debate.

The most common methods for exploring non-thermal femtomagnetism include the pump-probe technique and terahertz emission spectroscopy [1]. In typical optomagnetic

experiments, the light-induced oscillations of magnetization lasting a long time after termination of the subpicosecond pump laser pulse are detected. Measuring the aftereffects of the optical excitation rather than the transient magnetization induced via IFE during the action of the laser pulse provides only indirect information about ultrafast IFE. To clarify the nature of the ultrafast IFE, more direct measurements are required.

In recent paper [6], measuring terahertz Cherenkov radiation from a moving pulse of light-induced magnetization was proposed as a method to explore ultrafast optomagnetic phenomena. In this method, the circularly polarized laser pulse propagates in a slab of an optically transparent magneto-optic material, for example, terbium gallium garnet (TGG), along its surface and produces ultrafast magnetization via IFE. The moving magnetic moment generates a Cherenkov cone of terahertz waves in the output prism attached to the surface of the slab. Analysis of the terahertz waveform can provide valuable information about the ultrafast magnetization and, therefore, about IFE.

In the present paper, we implemented the scheme proposed in [6] and observed terahertz Cherenkov radiation from a moving magnetic moment. Our experimental setup is

schematically shown in Fig. 1. A Ti:sapphire-based amplified laser system (800 nm central wavelength, 25 fs pulse duration, 0.5 mJ pulse energy, and 3 kHz repetition rate) was used as a light source. The laser beam was attenuated and divided into a pump and probe beams. The polarization of the pump beam was controlled by a quarter wave plate. The pump beam was focused by a cylindrical lens to an entrance facet of a 2 mm thick TGG slab (10×10 mm<sup>2</sup> lateral dimensions). The slab was covered with a 35 µm thick lithium niobate (LN) layer, which was used for alignment of the pump and probe arms, and a high-resistivity-Si prism (41° apex angle). The focal spot of the pump beam was line-like with the dimensions 8 µm



 $\times$  6 mm. The terahertz radiation emitted from the Si prism was collected by the combination of plane and parabolic mirrors, and the terahertz waveform was detected using standard EO sampling technique in a 1 mm thick ZnTe crystal.

Fig. 2 (a) and (b) show the terahertz waveforms emitted from the sandwich structure for two different circular polarizations of the pump beam (for a linear polarization, there was no signal). The width of the terahertz spectrum [Fig. 2 (c)] agrees well with the pump pulse duration 500 fs measured after the crystal.

By analyzing the polarity of the terahertz pulses we conclude that the Verdet constant in TGG is negative, i.e., the ultrafast magnetization has paramagnetic nature, contrary to some theoretical speculations [7].

The magnitude of the Verdet constant in the subpicosecond regime is estimated to be  $\sim 3\div 10$  times smaller than its table quasistatic value. This result is in striking contrast with the results of the pump-probe measurements published recently in [7]. According to [7], the Verdet constant is 4 orders of magnitude larger than its quasistatic value. If it were true, we would observe a very strong signal from TGG – three orders of magnitude stronger than the signal measured in our experiment.

To conclude, the Cherenkov-type terahertz emission spectroscopy allows one to clarify the mechanism of the ultrafast IFE, in particular, in paramagnetic materials.



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